

# Teletraffic Model for K-fold Multicast Network: Performance Evaluation with Buffer using M/M/n/n+q/N Kendal's Notation

Md. Mahmudul Hasan\* & Most. Tajmary Mahfuz\*\*

\*Game Intelligence Group, eProfessionals.org.uk; Sr. Lecturer, Department of Computer Science and Engineering, Daffodil International University, Dhaka, BANGLADESH. Email: mahmudul.05@gmail.com

\*\*Lecturer, Department of Natural Science, Daffodil International University, Dhaka, BANGLADESH. Email: tajmary@daffodilvarsity.edu.bd

**Abstract**—This paper is an extended work of Hasan et al., (2012) where a teletraffic model was shown. In a previous research work, the authors presented a k fold multicast network with optimum output. Multicast has brought a change in network operation by providing bandwidth effective technology and reducing traffic loads. It is popular for performance optimization in a network. In this paper, a new model has been designed to establish a suitable value of the fold k of a k-fold multicast network under different traffic loads under Poisson traffic with finite queue at each node. Stationary distribution has been derived for the network states with the derivation of expressions for the network throughput and the blocking probability of the network. The research has explored that the network throughput increases very fast if the fold number is increased. However, at a certain value of the fold, the blocking probability stops to increase and it becomes constant. The study further discovered that as the offered traffic is increased, the throughput also increases. With the increase of system parameter k, the blocking probability decreases. Moreover, after an optimum value of k, the blocking probability becomes static for a particular value of the offered traffic.

**Keywords**—K-fold Network; Kendal's Notation; Markov Chain; Multicasting; Teletraffic; Throughput; Traffic Theory.

**Abbreviations**—Broad-band Integrated Services Network (BISDN); Mutually Compatible (m.c); Quality of Service (QoS).

## I. INTRODUCTION

IN this research work, we have shown teletraffic model and its impact for k-fold multicast network whereas in the last work, we have shown only optimization of k fold multicast network with finite queue which has been published in a conference named 7<sup>th</sup> International Conference on Electrical and Computer Engineering (ICECE-2012) [Hasan et al., 2012].

Multicast involves transmitting information from a single source to multiple destinations. This is an important requirement for high-performance communications networks. Multicast communication is one of the most important collective communication operations and is highly demanded in Broad-band Integrated Services Network (BISDN) and in communication-intensive applications in parallel and distributed computing systems, such as distributed database updates and cache coherence protocols. It is projected that

multicast will also be increasingly used to support various other interactive applications such as multimedia, teleconferencing, web servers and electronic commerce on the Internet [Md. Mahmudul Hasan, 2013]. Many of these applications require predictable communications performance, such as guaranteed multicast latency and bandwidth, called Quality of Service (QoS) in addition to multicast capability. The QoS guarantees and the non-uniform nature of multicast traffic make the problem of the analysis of multicast communication is very challenging.

However, to provide a quantitative basis for the network designers, determining an optimum value of the system parameter  $k$  (the fold number) is essential. Keeping this view in mind, we have, in this research work, developed an analytical model to determine the suitable value of  $k$  under different traffic loads for a  $k$ -fold multicast network under Poisson traffic with finite buffers or queue at each node [Md. Syeful Islam et al., 2011]. We have derived stationary

distribution for the network states and then derived expressions for the network throughput and the blocking probability of the network. Moreover, in this paper we have shown the adjustable parameter K with finite users using Markovian model M/M/n/n+q/N.

## II. LITERATURE REVIEW

### 2.1. Background

Multicasting is a technical term that is used as a networking technique of delivering messages and information to a group simultaneously from the source. A typical multicasting service is shown in the figure 1.

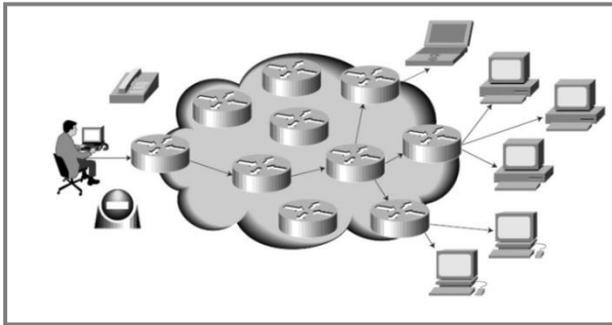


Figure 1: Basic Multicast Service

In a K-fold multicast network, fold number indicates the number of request coming from different sources to a particular destination. On the other hand, finite queue is a data set shared by program processes which acts as a buffer for data in multicast network. In this research work, we have developed a model to help network engineers to design an effective multicast network. To do this, it is necessary to get the optimum value of K in a k-fold multicast network. By implementing the optimum value of the system parameter k in k-fold multicast network, we improved the network performance by optimizing network throughput, where throughput is the number of messages successfully delivered per unit time. In this paper, the term “throughput” has been used to measure from the arrival of the first bit of data at the receiver [Anupam Roy et al., 2011].

As mentioned earlier, the primary target of this research work was to evaluate the performance of k-fold multicast network by using traffic model. For this reason, throughout this paper, we make following assumptions on the multicast traffic we consider.

- The probability of a destination node being involved in an incoming multicast connection request is independent of other destination nodes.
- Multicast connection requests at different source nodes are independent to each other.
- Holding time of each multicast connection is exponentially distributed with parameter and is independent to each other.
- Multicast connection requests arrive at each source node according to a Poisson process with intensity and are independent to each other [Zhenghao Zhang & Yang, 2005; Abu Sayed Rahaman et al., 2011].

### 2.2. Previous Researches

This research work is basically the extension of previous works of Hsan et al., (2012), Zhenghao Zhang & Yang (2005) who evaluated the performance of k fold network but they did not use buffers. After that Asfara R. Rafiq et al., (2008) again checked the performance of K-fold network with a new look. They show the optimization of K-fold multicast network with buffers but for infinite users. Here, we have used finite users to evaluate performance of k-fold multicast network by using Markovian model.

For this approach, a destination node may be simultaneously involved in two multicast connections. Such connections will be blocked in a network which is designed to be non-blocking or rearrange able for only multicast assignments. Specifically, the network can realize multiple multicast assignments in a single pass with a guaranteed latency.

## III. TRAFFIC THEORY AND K-FOLD MULTICAST NETWORK

### 3.1. Basic Traffic Theory and Markov Chain

Traffic Theory describes the key models of traffic flow and associated traffic phenomena such as conflicts in traffic, congestion control and effective management of traffic.

In this paper, we have derived stationary distribution of the k-fold network from which we can obtain network throughput and the blocking probability. We assume the Markovian M/M/n/n+q/N model which is shown in figure 2.

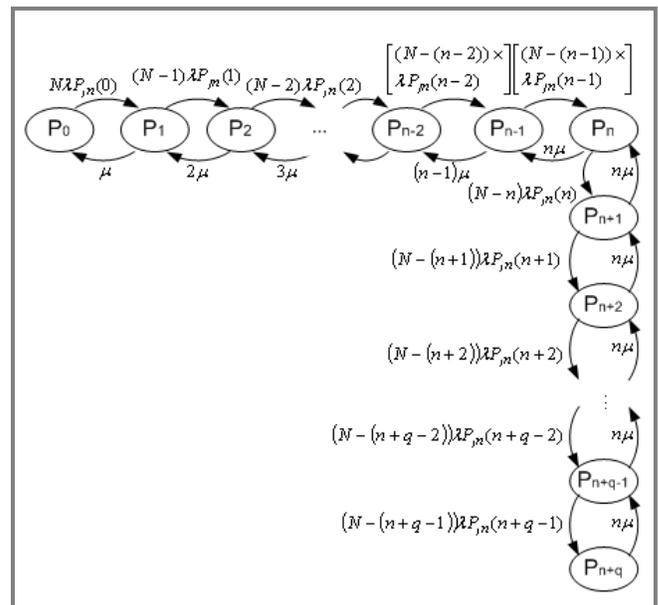


Figure 2: Markov Chain for K-fold Network

The above figure shows a glance of Markov Chain and its impact on Finite State space.

### 3.2. K-fold Network

It is defined as a mapping from a subset of network source nodes to a subset of network destination nodes, with up to K-

fold overlapping allowed among the destinations of different sources. It is an adjustable parameter. In other words, any destination node can be involved in multicast connections from up to K different sources at a time [Asfara R. Rafiq et al., 2008].

*Why a K-fold Network?*

- A cost-effective solution to provide better quality-of-service functions in supporting real-world multicast applications.
- Predictable communications performance, such as guaranteed multicast latency and bandwidth.
- Highly demanded in communication-intensive applications in parallel and distributed computing systems, such as distributed database updates.

**3.3. Kendal’s Notation of Queuing System**

In 1953 D.G Kendall introduces special notation for queuing models. A complete notation for the paper is:

$$M/M/n/K/N$$

Where,

M: Markov or memory less which follows exponential distribution

N: Number of servers/channels

N: Number of users

K=n+q: Sum of channels and queue

q: Length of queue

**IV. MATHEMATICAL ANALYSIS**

Let us consider that there are  $j$  multicast connection requests [Hasan et al., 2012A], and let  $P_{deg}(j, m)$  be the probability that a destination node is the destination of exactly  $m$  of the multicast connection requests; or we can say that a destination node is of degree  $m$  under these  $j$  multicast connection requests. The probability that any multicast connection request chooses this destination node is  $\theta$  and is independent of other multicast connections. Thus, we have

$$P_{deg}(j, m) = \binom{j}{m} \theta^m (1-\theta)^{j-m}, \quad m \in \{0, 1, \dots, j\} \quad (1)$$

which is a binomial random variable. We assume that each destination node has the same distribution given by (1).

Furthermore, we assume that whether a destination node is chosen by a multicast connection is independent of other destination nodes. Thus, in addition to having the same distributions, the degrees of the destination nodes are also independent of each other. That is they are a group of independent, identically distributed (i.i.d.) random variables [Feldman et al., 1988].

**4.1. Mathematical Analysis of Network Throughput**

Let  $P_{mc}(j)$  be the probability that  $j$  multicast connection requests are mutually compatible (*m.c*) in a  $k$ -fold multicast network. We note that a set of multicast connection requests is *m.c*. When none of the destination nodes has a degree more than  $k$  when realized simultaneously in the network. From

(1), it is obvious that the probability of a destination node having a degree less than or equal to  $k$  is  $\sum_{m=0}^k P_{deg}(j, m)$  for  $j > k$ , and 1 for  $j \leq k$ , because when  $j \leq k$ , no destination node can have a degree more than  $k$ . Since the degrees of destination nodes are independent of each other, we have

$$P_{mc}(j) = \begin{cases} \left( \sum_{m=0}^k P_{deg}(j, m) \right)^n, & j > k \\ 1, & \text{otherwise.} \end{cases} \quad (2)$$

Now, let us consider that a new multicast connection request arrives when  $j$  multicast connections are already in the network. If this new connection can be realized along with those ongoing connections, we say that it can *join* the ongoing connections. Let  $P_{jn}(j)$  be the probability that a new multicast connection can join  $j$  ongoing connections. It can be shown that

$$P_{jn}(j) = \frac{P_{mc}(j+1)}{P_{mc}(j)}. \quad (3)$$

By solving the Markov chain of Figure 2, the stationary states are found to have the probabilities

$$P_r = \binom{N}{r} \rho^r \prod_{x=0}^{r-1} P_{jn}(x) P_0, \quad 0 \leq r \leq n$$

Total number of times the network departs from state  $i$  due to the arrival of a successful connection request is,

$$TP_j(n + q - j) \lambda P_{jn}(j)$$

This is also the total numbers of successful connection requests among at the network when the network is in state  $j$  ( $j \in \{0, 1, \dots, N\}$ ) during  $[0, T]$ . Therefore the total number of successful connection requests carried by the network during  $[0, T]$  is obtained by summing average,

$$N_{succ} = T \lambda \sum_{j=0}^{n+q} P_j(n + q - j) P_{jn}(j)$$

Therefore the network Throughput is,

$$T_H = \frac{N_{succ}}{T} = \lambda \sum_{j=0}^{n+q} p_j P_{jn}(j) (n + q - j) \quad (4)$$

**4.2. Mathematical Analysis of Blocking Probability**

The total number of connection requests arriving at the network during  $[0, T]$

$$N_{total} = (n + q) \lambda T$$

Thus the Blocking Probability,

$$P_B = \frac{N_{b1}}{N_{total}} = \left[ 1 - \frac{1}{n + q} \sum_{j=0}^{n+q} (n + q - j) P_{jn}(j) P_j \right]$$

**4.3. Mathematical Analysis of Probability of Delay**

The Probability of Delay is,

$$P_D = \sum_{s=1}^q P_{n+s} = \sum_{s=1}^q (n + s)! \frac{P^n}{n!} \binom{P}{n}^s \binom{N}{n+s} \prod_{l=0}^{n+s-1} p_{jn}(l) P_0 \quad (5)$$

### V. RESULTS AND DISCUSSIONS

For numerical appreciation of our results, we have plotted in Figures (3), (4) and (5), the throughput and the blocking probability as a function of the fold number  $k$  [Feldman et al., 1988].

It is seen from Figure 3 that if the fold of the network is increased, network throughput increases very fast in the lower values of the system parameter  $k$ , in our study up to  $k = 5$ ; beyond this value of  $k$ , the network throughput is almost constant with respect to the system parameter  $k$  for particular offered traffic. We also observe that as the offered traffic is increased, the throughput also increases [McKeown et al., 1999; Andrews et al., 1999].

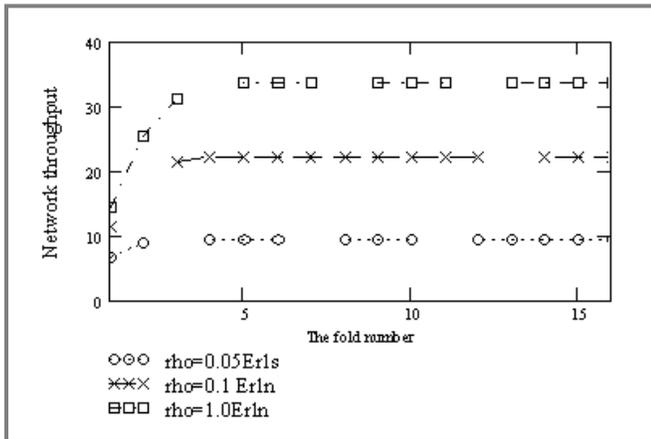


Figure 3: Network Throughput as a Function of the Fold Number under Different Offered Traffic ( $N=50, n=14, q=5, \theta=.31$ )

Figure 4 shows the variation of the blocking probability with respect to the fold  $k$ . It is seen from this figure that as the system parameter  $k$  increases, the blocking probability decreases. However, after an optimum value of  $k$ , in our present study it is  $\sim 5$ , the blocking probability remains constant for particular value of the offered traffic.

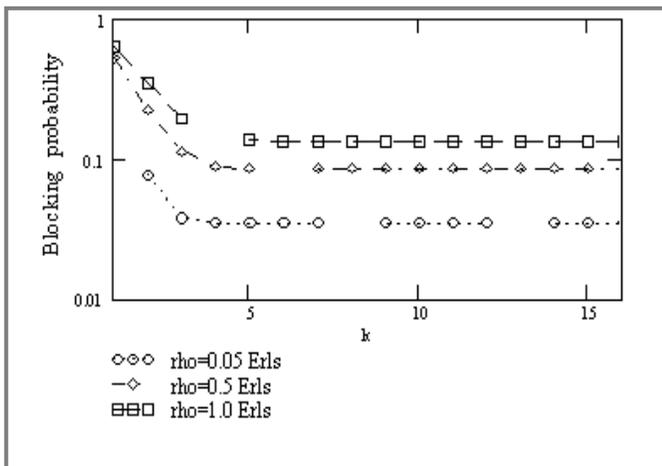


Figure 4: Blocking Probability as a Function of the Fold Number under Different Offered Traffic ( $N=50, n=14, q=5, \theta=.31$ )

Figure 5 shows, the variation of the probability of delayed service with respect to the fold number  $k$ . It is observed that the probability of delay is almost negligible for

lower values of the fold number  $k$ , whereas, it is suddenly increases as the fold number approaches the optimum value  $k \sim 5$ . However, after a certain value of  $k$ , the probability of delay becomes constant [Yang & Masson, 1991; Yang & Wang, 1998; 2003].

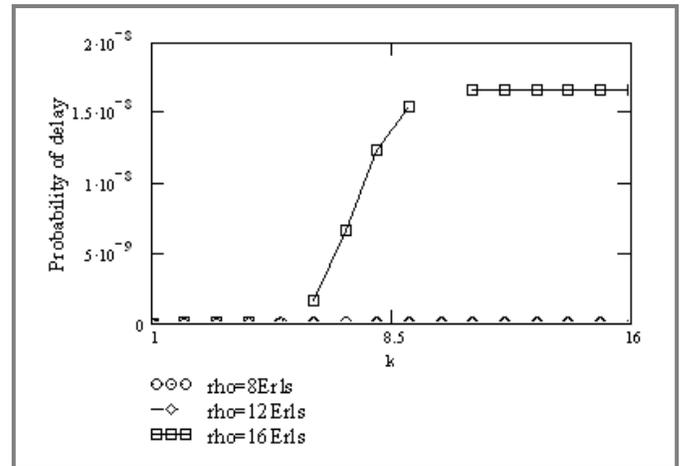


Figure 5: Probability of delay as a function of the fold number under different offered traffic ( $N=50, n=14, q=5, \theta=.31$ )

### VI. CONCLUSIONS

Determining the system parameter  $k$  (the fold number) and finding its optimum value is must to design an effective network. To keep this view in mind, we have developed a systematical model to determine an optimum value for a  $k$ -fold multicast network under Poisson traffic with finite queue at each node. We have derived stationary distribution for the network states and then derived expressions for the network throughput and the blocking probability of the network. It has been found in this study that the network throughput increases very fast as we increase the fold number. However, at a certain value of the fold, the blocking probability ceases to increase and it remains constant. We have also observed that as the offered traffic is increased, the throughput also increases.

In addition, it has been observed that the blocking probability decreased proportionately based on the traffic when the system parameter  $k$  in a  $k$ -fold multicast network is increased. However, after a suitable value of  $k$ , which is approximately 5 based on our research in this paper, the blocking probability remains constant for specific value of the offered traffic in multicast network.

Note that although  $K$ -fold multicast assignments can be realized by simply stacking  $k$  copies of one fold network together, the  $k$ -fold network designed in has a much lower hardware cost. In fact, the cost of the former is about 3- $k$  times of a  $k$ -fold network for any  $k$ . Thus, a  $k$ -fold network is a cost effective choice to provide better QoS functions in supporting arbitrary multicast communication.

To sum up, this model could help to find more suitable and appropriate value of the system parameter  $k$  in a  $k$ -fold multicast network to increase throughput of the network for designing an effective ubiquitous network in future.

## ACKNOWLEDGMENT

Authors of this paper would like to thank Prof. Dr. Md. Imdadul Islam, Department of Computer Science and Engineering, Jahangirnagar University, Savar, Dhaka for his helpful guidance in this research work. This research work is partially funded by eProfessionals.org.uk.

## REFERENCES

- [1] P. Feldman, J. Friedman & N. Pippenger (1988), "Wide-Sense Nonblocking Networks", *SIAM Journal on Discrete Mathematics*, Vol. 1, No. 2, Pp. 158–173.
- [2] Y. Yang & G.M. Masson (1991), "Nonblocking Broadcast Switching Networks", *IEEE Transactions on Computers*, Vol. 40, Pp. 1005–1015.
- [3] Y. Yang & J. Wang (1998), "On Blocking Probability of Multicast Networks", *IEEE Transactions on Communications*, Vol. 46, Pp. 957–968.
- [4] M. Andrews, S. Khanna & K. Kumaran (1999), "Integrated Scheduling of Unicast and Multicast Traffic in an Input-Queued Switch", *Proceedings of IEEE INFOCOM*, Pp. 1144–1151.
- [5] N. McKeown, A. Mekkittijul, V. Anantharam & J. Walrand (1999), "Achieving 100% Throughput in an Input-Queued Switch", *IEEE Transactions on Communications*, Vol. 47, Pp. 1260–1267.
- [6] Y. Yang & J. Wang (2003), "Nonblocking K-fold Multicast Networks", *IEEE Transactions on Parallel and Distributed Systems*, Vol. 14, Pp. 131–141.
- [7] Zhenghao Zhang & Y. Yang (2005), "Performance Analysis of K-fold Multicast Networks", *IEEE Transactions on Communications*, Vol. 53, Pp. 308–314.
- [8] Asfara R. Rafiq, N.A. Siddiky, Md. Imdadul Islam, M.R. Amin (2008), "Optimization of K-fold Wireless Network using M/M/n/n+q Traffic Model", *International Conference on Electrical and Communication Engineering*, Buet, Dhaka, Paper ID ICECE 125.
- [9] Abu Sayed Rahaman, Md. Imdadul Islam & M.R. Amin (2011), "Performance Evaluation of Two-Hop Wireless Link under Rayleigh and Nakagami-m Fading Channel", *International Journal of Engineering and Technology (IJET)*, (Singapore), Vol. 3, No. 5, Pp. 454–459.
- [10] Md. Syeful Islam, Md. Rezaur Rahman, Anupam Roy, Md. Imdadul Islam & M.R. Amin (2011), "Performance Evaluation of a Finite Queue Switching under Two-Dimensional M/G/1(m) Traffic", *International Journal of Imaging Systems (IJIS)*, Korea, Vol. 7, No. 4, Pp. 679–690.
- [11] Anupam Roy, Md. Imdadul Islam & M.R. Amin (2011), "MMPP+M/D/1 Traffic Model in Video-Data Integrated Service under ATM System", *International Journal of Engineering and Technology (IJET)*, Singapore, Vol. 3, No. 6, Pp. 615–620.
- [12] M.M. Hasan, M.T. Mahfuz & M.R. Amin (2012), "Optimizing throughput of K-fold Multicast Network with Finite Queue using M/M/n/n+q/N Traffic Model", *7th International Conference on Electrical & Computer Engineering (ICECE)*, Pp. 537–541.
- [13] MM Hasan, Most Tajmary Mahfuz, Woakil Uddin Ahamed, Syed Akhter Hossain (2012A), "Requirement Analysis for Context-Awareness to Create a Digital Persona in eHealth based Pervasive Intelligence Environment", *Journal of Information and Communication Technologies*, Vol. 2, No. 3, Pp.11–15.
- [14] Md. Mahmudul Hasan (2013), "Fine-Tuning of k in a K-fold Multicast Network with Finite Queue using Markovian Model", *International Journal of Computer Networks and Communications (IJCNC)*, Vol. 5, No. 2, Pp. 195–204.



**Md. Mahmudul Hasan** is currently serving himself as a sr. lecturer in the department of CSE, Daffodil International University, Bangladesh and as a chief architect of Game Intelligence Group at eProfessionals.org.uk. Born in 1987 at Pabna, Bangladesh, M. M. Hasan had outstanding performance in his early academic life. He completed his MSc in Computer Science from University of Essex (UoE), UK and worked as a research assistant in International Development Academy, UoE, UK. He achieved distinction in Pervasive Computing and Ambient Intelligence in his MSc. His research interests include data mining, digital persona, machine learning, gaming and so on.



**Most. Tajmary Mahfuz** is working as a lecturer in the department of Natural Science, Daffodil International University and has completed her MS in Statistics from Jahangirnagar University, Dhaka. She has strong background in field level researching and served herself as a Survey Research Analyst in the project of SME foundation under Maxwell Stamp Ltd. She also has experiences on survey-based project such as School Mapping of IFC contracted by ADSL.